6.6 Use Proportionality Theorems

Before You used proportions with similar triangles.

Now You will use proportions with a triangle or parallel lines.

Why? So you can use perspective drawings, as in Ex. 28.

Key Vocabulary

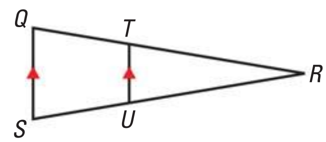
- **corresponding angles,** *p. 147*
- **ratio,** *p. 356*
- **proportion,** *p. 358*

The Midsegment Theorem, which you learned on page 295, is a special case of the Triangle Proportionality Theorem and its converse.

THEOREMS *For Your Notebook*

THEOREM 6.4 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

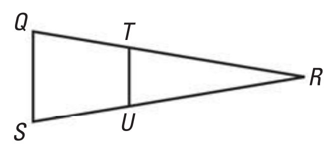


If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof: Ex. 22, p. 402

THEOREM 6.5 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

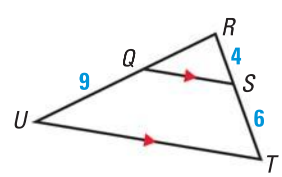


If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Proof: Ex. 26, p. 402

EXAMPLE 1 Find the length of a segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?



Solution

$\frac{RQ}{QU} = \frac{RS}{ST}$ Triangle Proportionality Theorem

$\frac{RQ}{9} = \frac{4}{6}$ Substitute.

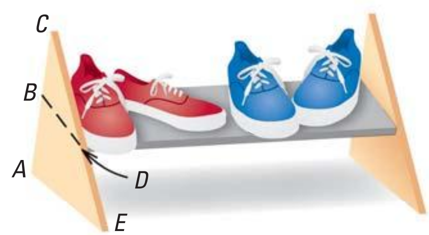
$RQ = 6$ Multiply each side by 9 and simplify.

REASONING Theorems 6.4 and 6.5 also tell you that if the lines are *not* parallel, then the proportion is *not* true, and vice-versa.

So if $\overline{TU} \nparallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$. Also, if $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \nparallel \overline{QS}$.

EXAMPLE 2 Solve a real-world problem

SHOERACK On the shoerack shown, $AB = 33$ cm, $BC = 27$ cm, $CD = 44$ cm, and $DE = 25$ cm. *Explain* why the gray shelf is not parallel to the floor.



Solution

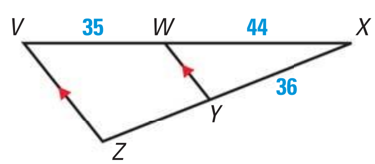
Find and simplify the ratios of lengths determined by the shoerack.

$$\frac{CD}{DE} = \frac{44}{25} \qquad \frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$$

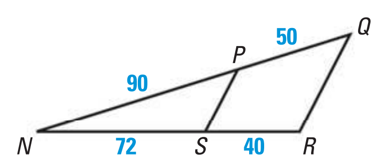
▶ Because $\frac{44}{25} \neq \frac{9}{11}$, $\overline{BD}$ is not parallel to $\overline{AE}$. So, the shelf is not parallel to the floor.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the length of $\overline{YZ}$.



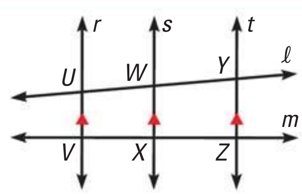
2. Determine whether $\overline{PS} \parallel \overline{QR}$.



THEOREMS *For Your Notebook*

THEOREM 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

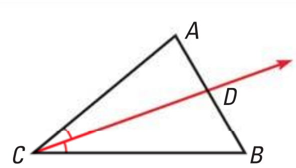


$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Proof: Ex. 23, p. 402

THEOREM 6.7

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

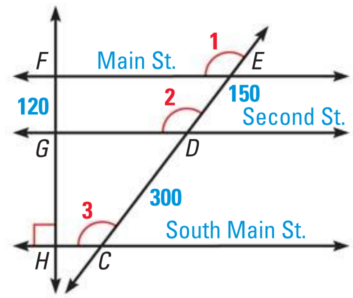


$$\frac{AD}{DB} = \frac{CA}{CB}$$

Proof: Ex. 27, p. 403

EXAMPLE 3 Use Theorem 6.6

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



ANOTHER WAY
For alternative methods for solving the problem in Example 3, turn to page 404 for the **Problem Solving Workshop**.

Solution

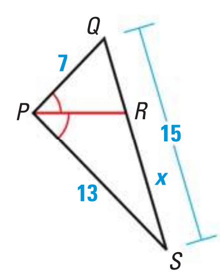
Corresponding angles are congruent, so $\overleftrightarrow{FE}$, $\overleftrightarrow{GD}$, and $\overleftrightarrow{HC}$ are parallel. Use Theorem 6.6.

$\frac{HG}{GF} = \frac{CD}{DE}$	**Parallel lines divide transversals proportionally.**
$\frac{HG + GF}{GF} = \frac{CD + DE}{DE}$	**Property of proportions (Property 4)**
$\frac{HF}{120} = \frac{300 + 150}{150}$	**Substitute.**
$\frac{HF}{120} = \frac{450}{150}$	**Simplify.**
$HF = 360$	**Multiply each side by 120 and simplify.**

▶ The distance between Main Street and South Main Street is 360 yards.

EXAMPLE 4 Use Theorem 6.7

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.



Solution

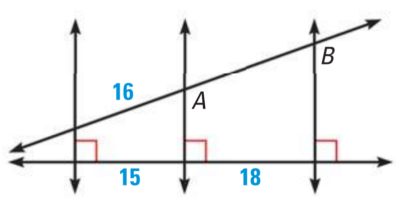
Because $\overrightarrow{PR}$ is an angle bisector of $\angle QPS$, you can apply Theorem 6.7. Let $RS = x$. Then $RQ = 15 - x$.

$\frac{RQ}{RS} = \frac{PQ}{PS}$	**Angle bisector divides opposite side proportionally.**
$\frac{15 - x}{x} = \frac{7}{13}$	**Substitute.**
$7x = 195 - 13x$	**Cross Products Property**
$x = 9.75$	**Solve for *x*.**

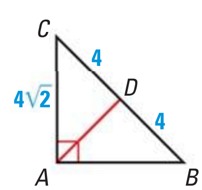
✓ GUIDED PRACTICE for Examples 3 and 4

Find the length of $\overline{AB}$.

3.



4.



6.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 13, 25, and 28

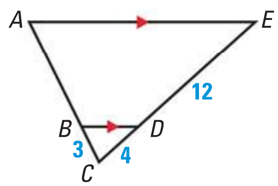
SKILL PRACTICE

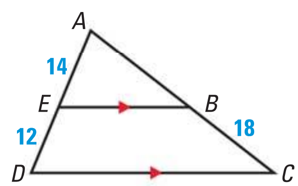
1. **VOCABULARY** State the Triangle Proportionality Theorem. Draw a diagram.

2. ★ **WRITING** *Compare* the Midsegment Theorem (see page 295) and the Triangle Proportionality Theorem. How are they related?

EXAMPLE 1 on p. 397 for Exs. 3–4

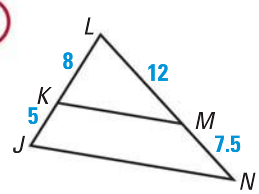
FINDING THE LENGTH OF A SEGMENT **Find the length of $\overline{AB}$.**

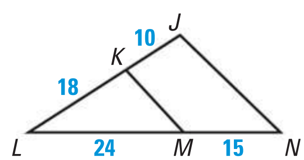
3. 

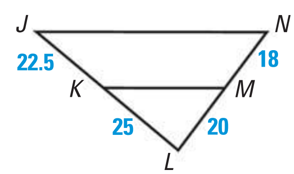
4. 

EXAMPLE 2 on p. 398 for Exs. 5–7

REASONING **Use the given information to determine whether $\overline{KM} \parallel \overline{JN}$. *Explain* your reasoning.**

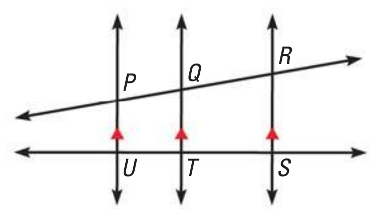
5. 

6. 

7. 

EXAMPLE 3 on p. 399 for Ex. 8

8. ★ **MULTIPLE CHOICE** For the figure at the right, which statement is *not* necessarily true?

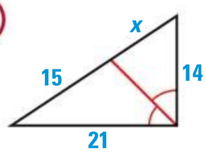


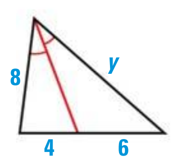
Ⓐ $\frac{PQ}{QR} = \frac{UT}{TS}$ Ⓑ $\frac{TS}{UT} = \frac{QR}{PQ}$

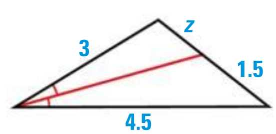
Ⓒ $\frac{QR}{RS} = \frac{TS}{RS}$ Ⓓ $\frac{PQ}{PR} = \frac{UT}{US}$

EXAMPLE 4 on p. 399 for Exs. 9–12

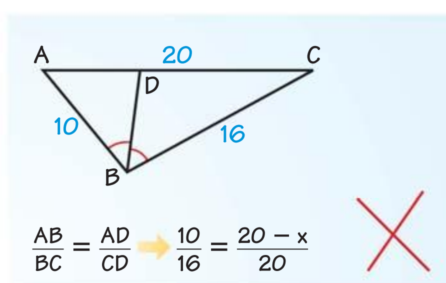
ALGEBRA **Find the value of the variable.**

9. 

10. 

11. 

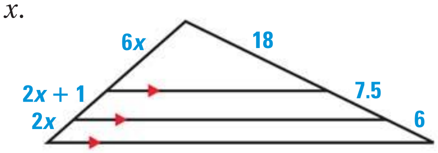
12. **ERROR ANALYSIS** A student begins to solve for the length of $\overline{AD}$ as shown. *Describe* and correct the student's error.



13. ★ **MULTIPLE CHOICE** Find the value of x.

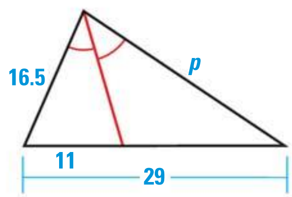
Ⓐ $\frac{1}{2}$ Ⓑ 1

Ⓒ 2 Ⓓ 3

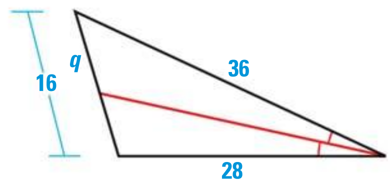


ALGEBRA **Find the value of the variable.**

14.

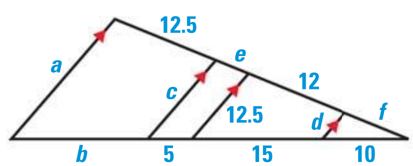


15.

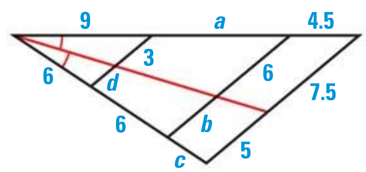


FINDING SEGMENT LENGTHS **Use the diagram to find the value of each variable.**

16.

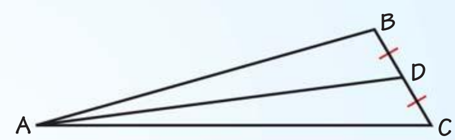


17.



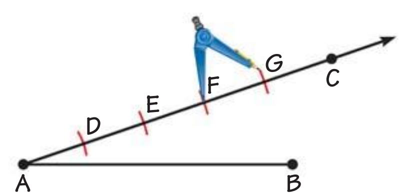
18. **ERROR ANALYSIS** A student claims that $AB = AC$ using the method shown. *Describe* and correct the student's error.

By Theorem 6.7, $\frac{BD}{CD} = \frac{AB}{AC}$. Because $BD = CD$, it follows that $AB = AC$.



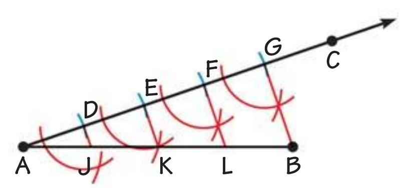
19. **CONSTRUCTION** Follow the instructions for constructing a line segment that is divided into four equal parts.

a. Draw a line segment that is about 3 inches long, and label its endpoints A and B. Choose any point C not on $\overline{AB}$. Draw $\overrightarrow{AC}$.

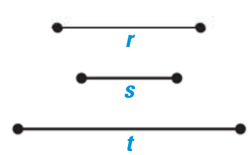


b. Using any length, place the compass point at A and make an arc intersecting $\overrightarrow{AC}$ at D. Using the same compass setting, make additional arcs on $\overrightarrow{AC}$. Label the points E, F, and G so that $AD = DE = EF = FG$.

c. Draw $\overline{GB}$. Construct a line parallel to $\overline{GB}$ through D. Continue constructing parallel lines and label the points as shown. *Explain* why $AJ = JK = KL = LB$.

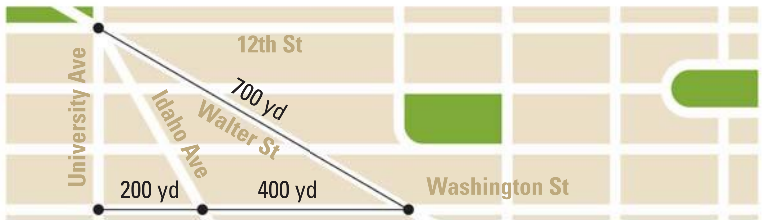


20. **CHALLENGE** Given segments with lengths r, s, and t, construct a segment of length x, such that $\frac{r}{s} = \frac{t}{x}$.



PROBLEM SOLVING

21. **CITY MAP** On the map below, Idaho Avenue bisects the angle between University Avenue and Walter Street. To the nearest yard, what is the distance along University Avenue from 12th Street to Washington Street?



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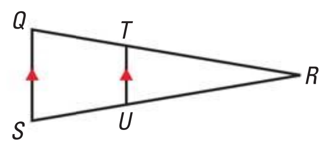
22. **PROVING THEOREM 6.4** Prove the Triangle Proportionality Theorem.

GIVEN ▶ $\overline{QS} \parallel \overline{TU}$

PROVE ▶ $\frac{QT}{TR} = \frac{SU}{UR}$



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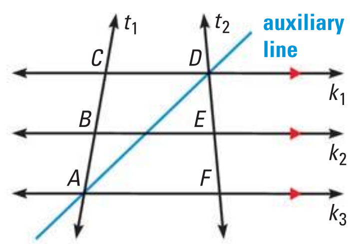
23. **PROVING THEOREM 6.6** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 6.6.

GIVEN ▶ $k_1 \parallel k_2$, $k_2 \parallel k_3$

PROVE ▶ $\frac{CB}{BA} = \frac{DE}{EF}$

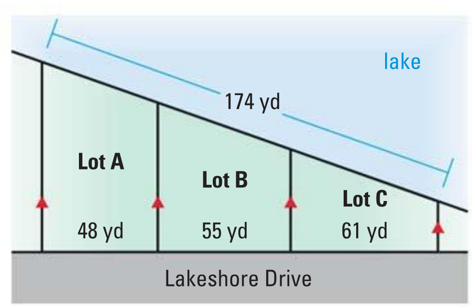


24. **MULTI-STEP PROBLEM** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.

a. Find the lake frontage (to the nearest tenth of a yard) for each lot shown.

b. In general, the more lake frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?

c. Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is $100,000, what are the prices of the other lots? *Explain* your reasoning.



25. ★ **SHORT RESPONSE** Sketch an isosceles triangle. Draw a ray that bisects the angle opposite the base. This ray divides the base into two segments. By Theorem 6.7, the ratio of the legs is proportional to the ratio of these two segments. *Explain* why this ratio is 1 : 1 for an isosceles triangle.

26. **PLAN FOR PROOF** Use the diagram given for the proof of Theorem 6.4 in Exercise 22 to write a plan for proving Theorem 6.5, the Triangle Proportionality Converse.

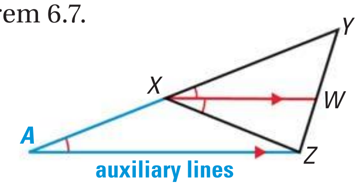
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

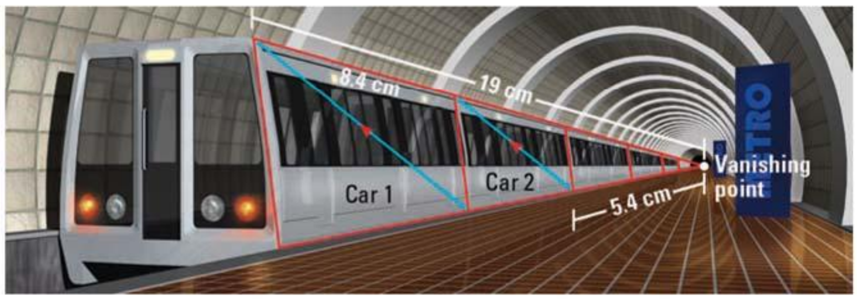
27. PROVING THEOREM 6.7 Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 6.7.

GIVEN ▶ $\angle YXW \cong \angle WXZ$

PROVE ▶ $\dfrac{YW}{WZ} = \dfrac{XY}{XZ}$



28. ★ EXTENDED RESPONSE In *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel.

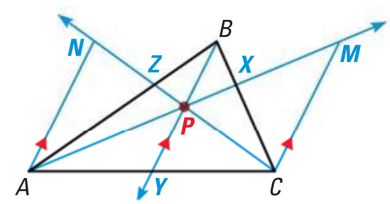


a. Use the dimensions given and the red parallel lines to find the length of the bottom edge of the drawing of Car 2.

b. What other set of parallel lines exist in the figure? *Explain* how these can be used to form a set of similar triangles.

c. Find the length of the top edge of the drawing of Car 2.

29. CHALLENGE Prove *Ceva's Theorem:* If P is any point inside $\triangle ABC$, then $\dfrac{AY}{YC} \cdot \dfrac{CX}{XB} \cdot \dfrac{BZ}{ZA} = 1$. (*Hint:* Draw lines parallel to $\overline{BY}$ through A and C. Apply Theorem 6.4 to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.)



MIXED REVIEW

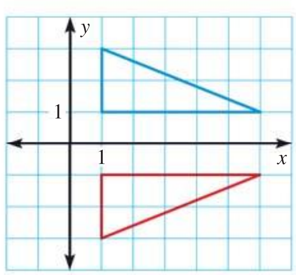
PREVIEW
Prepare for Lesson 6.7 in Exs. 30–36.

Perform the following operations. Then simplify.

30. $(-3) \cdot \dfrac{7}{2}$ *(p. 869)* **31.** $\dfrac{4}{3} \cdot \dfrac{1}{2}$ *(p. 869)* **32.** $5\left(\dfrac{1}{2}\right)^2$ *(p. 871)* **33.** $\left(\dfrac{5}{4}\right)^3$ *(p. 871)*

***Describe* the translation in words and write the coordinate rule for the translation.** *(p. 272)*

34.



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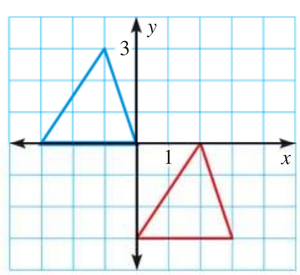

36.